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Scheme for Implementing Assisted Cloning of an Unknown Tripartite Entangled State

Jin Shi · You-Bang Zhan

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Abstract In this paper, we propose a protocol which can realize quantum cloning of an unknown tripartite entangled state and its orthogonal complement state with assistance from a state preparer. The first stage of the protocol requires usual teleportation via three entangled particle pairs as quantum channel. In the second stage of the protocol, the perfect copies and complement copies of an unknown state can be produced with the assistance (through a tripartite projective measurement) of the state preparer. We also present a scheme for the teleportation by using non-maximally entangled quantum channel. It is shown that the clones and complement clones of the unknown state can be obtained with certain probability in the latter scheme.

Keywords Quantum cloning · Tripartite entangled state · Tripartite projective measurement · Unitary transformation

1 Introduction

Quantum teleportation is a communication protocol for transmitting the state of a quantum system from one place to another without passing the system itself [1]. In recent years many theoretical and experimental researches have extensively explored teleportation from all aspects [2]. Several schemes for teleportation of a two-particle and multi-particle entangled state have been presented [3–10]. Recently, the possibility of cloning quantum states approximately has attracted extensive attentions [11]. Though exact cloning is no possible [12], in the thesis various cloning machines have been proposed [13–22] which operate either in a deterministic or probabilistic way. Universal quantum cloning machines was originally addressed by Bužek and Hillery [14]. The probabilistic cloning machine, proposed firstly by

J. Shi · Y.-B. Zhan
Department of Physics, Huaiyin Teachers College, Huaian 223001, Jiangsu, China

Y.-B. Zhan
CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
e-mail: ybzhan@hytc.edu.cn

Duan and Guo [16] using a general unitary-reduction operation with a postelection of the measurement results. Murao et al., [19] proposed the quantum telecloning process combining quantum teleportation and optimal quantum cloning from one input to M outputs. The other category of quantum cloning machines were developed by some authors. Pati [20] proposed a scheme where one can produce perfect copies and orthogonal-complement copies of an arbitrary unknown state with minimal assistance from a state preparer. Chen and Wu [15] presented a protocol which can probabilistically clone an unknown state and its orthogonal complement state with assistance. Zhan [22] further generalize and propose a protocol where one can realize quantum assisted cloning of an unknown bipartite entangled state and its orthogonal complement state with assistance offered by a state preparer.

In this paper, we propose a protocol which can realize assisted cloning of an unknown tripartite entangled state. Different from the previous protocols using single-particle von Neumann orthogonal measurement [15, 20] or bipartite projective measurement [22], here we will realize the assisted cloning via a three-particle projective measurement.

2 Assisted Cloning of an Unknown Tripartite Entangled State via Three Maximally Entangled Particle Pairs

Suppose Alice has an unknown input tripartite entangled state $|\phi\rangle_{123} = (\alpha|000\rangle + \beta|111\rangle)_{123}$ from a state preparer Victor, with α as a real number and β as a complex number and $|\alpha|^2 + |\beta|^2 = 1$. Alice wish to produce either a copy or an orthogonal copy of the unknown state $|\phi\rangle_{123}$ at her place with an assistance of Victor. Assume Alice and Bob share three entangled particle pairs as the quantum channel, which given by

$$\begin{aligned} |\psi\rangle_{45} &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{45}, \\ |\psi\rangle_{67} &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{67}, \\ |\psi\rangle_{89} &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{89}. \end{aligned} \tag{1}$$

Here we assume that particles 4, 6 and 8 belong to Alice while particles 5, 7 and 9 belongs to Bob. The initial state of the combined system is

$$|\Psi\rangle = |\phi\rangle_{123} \otimes |\psi\rangle_{45} \otimes |\psi\rangle_{67} \otimes |\psi\rangle_{89}. \tag{2}$$

In order to realize teleportation, Alice performs Bell measurements on particles (1, 4), (2, 6) and (3, 8), respectively. After the three measurements, all the possible outcomes are

$$\begin{aligned} {}_{38}\langle\Phi^{\pm}|_{26}\langle\Phi^{\pm}|_{14}\langle\Phi^{\pm}|\Psi\rangle &= \frac{1}{8}(\alpha|000\rangle \pm\pm\pm \beta|111\rangle)_{579}, \\ {}_{38}\langle\Phi^{\pm}|_{26}\langle\Phi^{\pm}|_{14}\langle\Psi^{\pm}|\Psi\rangle &= \frac{1}{8}(\alpha|100\rangle \pm\pm\pm \beta|011\rangle)_{579}, \\ {}_{38}\langle\Phi^{\pm}|_{26}\langle\Psi^{\pm}|_{14}\langle\Phi^{\pm}|\Psi\rangle &= \frac{1}{8}(\alpha|010\rangle \pm\pm\pm \beta|101\rangle)_{579}, \\ {}_{38}\langle\Psi^{\pm}|_{26}\langle\Phi^{\pm}|_{14}\langle\Phi^{\pm}|\Psi\rangle &= \frac{1}{8}(\alpha|001\rangle \pm\pm\pm \beta|110\rangle)_{579}, \end{aligned} \tag{3}$$

$$
\begin{aligned}
{}_{38}\langle\Psi^{\pm}|_{26}\langle\Phi^{\pm}|_{14}\langle\Psi^{\pm}|\Psi\rangle &= \frac{1}{8}(\alpha|101\rangle \pm\pm\pm \beta|010\rangle)_{579},\\
{}_{38}\langle\Phi^{\pm}|_{26}\langle\Psi^{\pm}|_{14}\langle\Psi^{\pm}|\Psi\rangle &= \frac{1}{8}(\alpha|110\rangle \pm\pm\pm \beta|001\rangle)_{579},\\
{}_{38}\langle\Psi^{\pm}|_{26}\langle\Psi^{\pm}|_{14}\langle\Phi^{\pm}|\Psi\rangle &= \frac{1}{8}(\alpha|011\rangle \pm\pm\pm \beta|100\rangle)_{579},\\
{}_{38}\langle\Psi^{\pm}|_{26}\langle\Psi^{\pm}|_{14}\langle\Psi^{\pm}|\Psi\rangle &= \frac{1}{8}(\alpha|111\rangle \pm\pm\pm \beta|000\rangle)_{579},
\end{aligned}
$$

where $|\Phi^{\pm}\rangle_{ij}$ and $|\Psi^{\pm}\rangle_{ij}$ are the Bell states of particles i and j

$$
\begin{aligned}
|\Phi^{\pm}\rangle_{ij} &= \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{ij},\\
|\Psi^{\pm}\rangle_{ij} &= \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{ij}.
\end{aligned}
\tag{4}
$$

In the above equations, the notes "±" from right to left correspond to signs of the Bell state of particles (1, 4), (2, 6) and (3, 8) that are shown in (3).

Assume Alice performs Bell-basis measurement on particles (1, 4), (2, 6) and (3, 8), respectively, and if the measurement outcome of Alice is $|\Phi^{-}\rangle_{14}|\Psi^{+}\rangle_{26}|\Psi^{+}\rangle_{38}$ (the probability of this result is only 1/64), then the resulting nine-particle state can be written as

$$
|\Psi^{+}\rangle_{38}\langle\Psi^{+}|\Psi^{+}\rangle_{26}\langle\Psi^{+}|\Phi^{-}\rangle_{14}\langle\Phi^{-}|\Psi\rangle = \frac{1}{8}|\Phi^{-}\rangle_{14}|\Psi^{+}\rangle_{26}|\Psi^{+}\rangle_{38}(\alpha|011\rangle - \beta|100\rangle)_{579}.
\tag{5}
$$

After these measurements, Alice sends the measurement result to Bob through a classical channel. According to the measure outcome of Alice, Bob will operate a unitary transformation $I_5 \otimes (\sigma_x)_7 \otimes (i\sigma_y)_9$ on (5), and to get the original state from particles 5, 7 and 9.

To create either a copy or an orthogonal-complement copy of the unknown tripartite state $|\phi\rangle$, Alice needs assistance of Victor. According to the projection postulate of quantum mechanics, if Alice applies projectors $|\Psi^{+}\rangle_{38}\langle\Psi^{+}|\Psi^{+}\rangle_{26}\langle\Psi^{+}|\Phi^{-}\rangle_{14}\langle\Phi^{-}|$ into the combined state $|\Psi\rangle$, the state of particles 1, 2, 3, 4, 6 and 8 will collapse in the entangled state $|\Phi^{-}\rangle_{14}|\Psi^{+}\rangle_{26}|\Psi^{+}\rangle_{38}$ (see (5)). Alice sends particles 1, 2, 3 to Victor and keeps particles 4, 6, 8 in her possession. Since Victor know the parameters α and β of original state $|\phi\rangle_{123}$ completely, he carries out a tripartite projective measurement on the particles 1, 2 and 3 in a set of mutually orthogonal basis vectors $\{|\varphi^{i}\rangle, |\varphi^{i}_{\perp}\rangle, (i = 0, 1, 2, 3)\}$, which is given by

$$
\begin{aligned}
|\varphi^{0}\rangle_{123} &= (\alpha|000\rangle + \beta|111\rangle)_{123},\\
|\varphi^{0}_{\perp}\rangle_{123} &= (\beta^{*}|000\rangle - \alpha|111\rangle)_{123},\\
|\varphi^{1}\rangle_{123} &= (\alpha|001\rangle + \beta|110\rangle)_{123},\\
|\varphi^{1}_{\perp}\rangle_{123} &= (\beta^{*}|001\rangle - \alpha|110\rangle)_{123},\\
|\varphi^{2}\rangle_{123} &= (\alpha|010\rangle + \beta|101\rangle)_{123},\\
|\varphi^{2}_{\perp}\rangle_{123} &= (\beta^{*}|010\rangle - \alpha|101\rangle)_{123},\\
|\varphi^{3}\rangle_{123} &= (\alpha|011\rangle + \beta|100\rangle)_{123},\\
|\varphi^{3}_{\perp}\rangle_{123} &= (\beta^{*}|011\rangle - \alpha|100\rangle)_{123}.
\end{aligned}
\tag{6}
$$

The above non-maximally entangled basis state $\{|\varphi^i\rangle, |\varphi^i_\perp\rangle, (i=0,1,2,3)\}$ are related to the computation basis vector $\{|000\rangle, |001\rangle, |010\rangle, |100\rangle, |110\rangle, |011\rangle, |101\rangle, |111\rangle\}$, and form a complete orthogonal basis in a eight-dimensional Hilbert space. We found that the $|\varphi^0\rangle_{123}$ is equal to $|\phi\rangle_{123}$ and the basis $|\varphi^0_\perp\rangle_{123}$ is equal to $|\phi_\perp\rangle_{123}$, where $|\phi_\perp\rangle_{123} = (\beta^*|000\rangle - \alpha|111\rangle)_{123}$ is the orthogonal-complement state to $|\phi\rangle_{123}$. Moreover, $|\varphi^1\rangle_{123} = (I_1 \otimes I_2 \otimes (\sigma_x)_3)|\phi\rangle_{123}$, $|\varphi^1_\perp\rangle_{123} = (I_1 \otimes I_2 \otimes (\sigma_x)_3)|\phi_\perp\rangle_{123}$ is the orthogonal-complement state to $|\varphi^1\rangle_{123}$. $|\varphi^2\rangle_{123} = (I_1 \otimes (\sigma_x)_2 \otimes I_3)|\phi\rangle_{123}$ and $|\varphi^2_\perp\rangle_{123} = (I_1 \otimes (\sigma_x)_2 \otimes I_3)|\phi_\perp\rangle_{123}$ is the orthogonal-complement state to $|\varphi^2\rangle_{123}$. And $|\varphi^3\rangle_{123} = (I_1 \otimes (\sigma_x)_2 \otimes (\sigma_x)_3)|\phi\rangle_{123}$ and $|\varphi^3_\perp\rangle_{123} = (I_1 \otimes (\sigma_x)_2 \otimes (\sigma_x)_3)|\phi_\perp\rangle_{123}$ is the orthogonal-complement state to $|\varphi^3\rangle_{123}$. (σ_x are Pauli operators). Thus, the entangled state $|\Phi^-\rangle_{14}|\Psi^+\rangle_{26}|\Psi^+\rangle_{38}$ in the basis $\{|\varphi^i\rangle, |\varphi^i_\perp\rangle, (i=0,1,2,3)\}$, can be rewritten as

$$\begin{aligned}
&|\Phi^-\rangle_{14}|\Psi^+\rangle_{26}|\Psi^+\rangle_{38} \\
&\quad= \frac{1}{2\sqrt{2}}[|\varphi^0\rangle_{123}(\alpha|011\rangle - \beta^*|100\rangle)_{468} + |\varphi^0_\perp\rangle_{123}(\alpha|100\rangle + \beta|011\rangle)_{468} \\
&\qquad + |\varphi^1\rangle_{123}(\alpha|010\rangle - \beta^*|101\rangle)_{468} + |\varphi^1_\perp\rangle_{123}(\alpha|101\rangle + \beta|010\rangle)_{468} \\
&\qquad + |\varphi^2\rangle_{123}(\alpha|001\rangle - \beta^*|110\rangle)_{468} + |\varphi^2_\perp\rangle_{123}(\alpha|110\rangle + \beta|001\rangle)_{468} \\
&\qquad + |\varphi^3\rangle_{123}(\alpha|000\rangle - \beta^*|111\rangle)_{468} + |\varphi^3_\perp\rangle_{123}(\alpha|111\rangle + \beta|000\rangle)_{468}].
\end{aligned} \tag{7}$$

If the result of Victor's measurement on the three particles 1, 2 and 3 is $|\varphi^0_\perp\rangle_{123}$, (5) can be written as

$$\begin{aligned}
&|\varphi^0_\perp\rangle_{123}\langle\varphi^0_\perp|\Psi^+\rangle_{38}\langle\Psi^+|\Psi^+\rangle_{26}\langle\Psi^+|\Phi^-\rangle_{14}\langle\Phi^-|\Psi\rangle \\
&\quad= \frac{1}{16\sqrt{2}}|\varphi^0_\perp\rangle_{123} \otimes ((\sigma_x)_4 \otimes I_6 \otimes I_8)|\phi\rangle_{468} \otimes (I_5 \otimes (\sigma_x)_7 \otimes (i\sigma_y)_9)|\phi\rangle_{579}.
\end{aligned} \tag{8}$$

Victor sends the measurement outcome to Alice through a classical channel, then Alice knows that the state of her particles 4, 6 and 8 has been found in the state $(\alpha|100\rangle + \beta|011\rangle)_{468}$, Alice will operate a unitary transformation $((\sigma_x)_4 \otimes I_6 \otimes I_8)$ on her particles 4, 6 and 8, and get a copy of the original state$|\phi\rangle_{123}$. If the result of Victor is $|\varphi^0\rangle_{123}$, (5) can be written as

$$\begin{aligned}
&|\varphi^0\rangle_{123}\langle\varphi^0|\Psi^+\rangle_{38}\langle\Psi^+|\Psi^+\rangle_{26}\langle\Psi^+|\Phi^-\rangle_{14}\langle\Phi^-|\Psi\rangle \\
&\quad= \frac{1}{16\sqrt{2}}|\varphi^0\rangle_{123} \otimes ((\sigma_x\sigma_z)_4 \otimes (\sigma_z)_6 \otimes I_8)|\phi_\perp\rangle_{468} \otimes (I_5 \otimes (\sigma_x)_7 \otimes (i\sigma_y)_9)|\phi\rangle_{579}.
\end{aligned} \tag{9}$$

Victor sends the measurement outcome to Alice through a classical channel, then Alice knows that the state of her particles 4, 6 and 8 has been found in the state $(\alpha|011\rangle - \beta^*|100\rangle)_{468}$, Alice will operate a unitary transformation $((\sigma_x\sigma_z)_4 \otimes (\sigma_z)_6 \otimes I_8)$ to get a complement copy of the original unknown state $|\phi\rangle_{123}$. If the results of Victor are $|\varphi^1_\perp\rangle_{123}$, $|\varphi^1\rangle_{123}$, $|\varphi^2_\perp\rangle_{123}$, $|\varphi^2\rangle_{123}$,$|\varphi^3_\perp\rangle_{123}$,$|\varphi^3\rangle_{123}$, (5) can be written as, respectively,

$$\begin{aligned}
&|\varphi^1_\perp\rangle_{123}\langle\varphi^1_\perp|\Psi^+\rangle_{38}\langle\Psi^+|\Psi^+\rangle_{26}\langle\Psi^+|\Phi^-\rangle_{14}\langle\Phi^-|\Psi\rangle \\
&\quad= \frac{1}{16\sqrt{2}}|\varphi^1_\perp\rangle_{123} \otimes ((\sigma_x)_4 \otimes I_6 \otimes (\sigma_x)_8)|\phi\rangle_{468} \otimes (I_5 \otimes (\sigma_x)_7 \otimes (i\sigma_y)_9)|\phi\rangle_{579},
\end{aligned}$$

$$
\begin{aligned}
&|\varphi^1\rangle_{123}\langle\varphi^1|\Psi^+\rangle_{38}\langle\Psi^+|\Psi^+\rangle_{26}\langle\Psi^+|\Phi^-\rangle_{14}\langle\Phi^-|\Psi\rangle\\
&\quad=\frac{1}{16\sqrt{2}}|\varphi^1\rangle_{123}\otimes(\sigma_x\sigma_z)_4\otimes I_6\otimes(i\sigma_y)_8)|\phi_\perp\rangle_{468}\otimes(I_5\otimes(\sigma_x)_7\otimes(i\sigma_y)_9)|\phi\rangle_{579},\\
&|\varphi_\perp^2\rangle_{123}\langle\varphi_\perp^2|\Psi^+\rangle_{38}\langle\Psi^+|\Psi^+\rangle_{26}\langle\Psi^+|\Phi^-\rangle_{14}\langle\Phi^-|\Psi\rangle\\
&\quad=\frac{1}{16\sqrt{2}}|\varphi_\perp^2\rangle_{123}\otimes((\sigma_x)_4\otimes(\sigma_x)_6\otimes I_8)|\phi\rangle_{468}\otimes(I_5\otimes(\sigma_x)_7\otimes(i\sigma_y)_9)|\phi\rangle_{579},\\
&|\varphi^2\rangle_{123}\langle\varphi^2|\Psi^+\rangle_{38}\langle\Psi^+|\Psi^+\rangle_{26}\langle\Psi^+|\Phi^-\rangle_{14}\langle\Phi^-|\Psi\rangle\\
&\quad=\frac{1}{16\sqrt{2}}|\varphi^2\rangle_{123}\otimes((i\sigma_y)_4\otimes(\sigma_x\sigma_z)_6\otimes I_8)|\phi_\perp\rangle_{468}\otimes(I_5\otimes(\sigma_x)_7\otimes(i\sigma_y)_9)|\phi\rangle_{579},\\
&|\varphi_\perp^3\rangle_{123}\langle\varphi_\perp^3|\Psi^+\rangle_{38}\langle\Psi^+|\Psi^+\rangle_{26}\langle\Psi^+|\Phi^-\rangle_{14}\langle\Phi^-|\Psi\rangle\\
&\quad=\frac{1}{16\sqrt{2}}|\varphi_\perp^3\rangle_{123}\otimes((\sigma_x)_4\otimes(\sigma_x)_6\otimes(\sigma_x)_8)|\phi\rangle_{468}\otimes(I_5\otimes(\sigma_x)_7\otimes(i\sigma_y)_9)|\phi\rangle_{579},\\
&|\varphi^3\rangle_{123}\langle\varphi^3|\Psi^+\rangle_{38}\langle\Psi^+|\Psi^+\rangle_{26}\langle\Psi^+|\Phi^-\rangle_{14}\langle\Phi^-|\Psi\rangle\\
&\quad=\frac{1}{16\sqrt{2}}|\varphi^3\rangle_{123}\otimes((\sigma_x\sigma_z)_4\otimes(i\sigma_y)_6\otimes(\sigma_x)_8)|\phi_\perp\rangle_{468}\otimes(I_5\otimes(\sigma_x)_7\otimes(i\sigma_y)_9)|\phi\rangle_{579}.
\end{aligned}
\tag{10}
$$

We can see that Alice gets a copy and a complement copy (all are up to doing a rotation operation) the unknown state, respectively. In the process of teleportation, if the Alice's measurement results are other 63 entangled states, from (3), applying the similar analysis as above, Alice will obtain a copy or a complement copy of the unknown state at her place.

3 Assisted Cloning of an Unknown Tripartite Entangled State via Three Non-Maximally Entangled States

Now, we propose another scheme for producing perfect copies of an unknown tripartite entangled state via three non-maximally entangled states as the quantum channel. Suppose that the unknown input state of Alice from Victor is still the tripartite entangled state $|\phi\rangle_{123}$, but the entangled particle pairs shared by Alice and Bob are three non-maximally entangled states, which are given by

$$
\begin{aligned}
|\psi'\rangle_{45}&=(a|01\rangle+b|10\rangle)_{45}, &&(|a|^2+|b|^2=1);\\
|\psi'\rangle_{67}&=(c|01\rangle+d|10\rangle)_{67}, &&(|c|^2+|d|^2=1);\\
|\psi'\rangle_{89}&=(e|01\rangle+f|10\rangle)_{89}, &&(|e|^2+|f|^2=1);
\end{aligned}
\tag{11}
$$

where a,b,c,d,e,f are real, $|a|\le|b|$, $|c|\le|d|$, $|e|\le|f|$. We assume Alice possesses particle 4, 6 and 8, Bob possesses particle 5, 7 and 9, $|\phi\rangle_{123}$ is still unknown to Alice and Bob. The combined nine-particle state is also expressed as

$$
|\Phi\rangle=|\phi\rangle_{123}\otimes|\psi'\rangle_{45}\otimes|\psi'\rangle_{67}\otimes|\psi'\rangle_{89}.
\tag{12}
$$

Taking the general case into account, Alice performs three measurement on particles (1, 4), (2, 6) and (3, 8) with the eigenstates [2]

$$
|\Phi_g^+\rangle_{ij}=(x|00\rangle+y|11\rangle)_{ij},
$$

$$|\Phi_g^-\rangle_{ij} = (y|00\rangle - x|11\rangle)_{ij},$$
$$|\Psi_g^+\rangle_{ij} = (x|10\rangle + y|01\rangle)_{ij}, \quad (13)$$
$$|\Psi_g^-\rangle_{ij} = (y|10\rangle - x|01\rangle)_{ij},$$

where x and y are real numbers, $|x|^2 + |y|^2 = 1$ and $|x| \le |y|$; $i, j = 1, 4$ and 2, 6 and 3, 8, respectively. Then Alice sends the measurement results to Bob through a classical channel. Assume Alice's result is $|\Phi_g^-\rangle_{14}|\Phi_g^+\rangle_{26}|\Psi_g^-\rangle_{38}$ then (12) can be written as

$$\begin{aligned} &|\Psi_g^-\rangle_{38}\langle\Psi_g^-|\Phi_g^+\rangle_{26}\langle\Phi_g^+|\Phi_g^-\rangle_{14}\langle\Phi_g^-|\Phi\rangle \\ &\quad = |\Phi_g^-\rangle_{14}|\Phi_g^+\rangle_{26}|\Psi_g^-\rangle_{38}(-x^2y\alpha acf|110\rangle - xy^2\beta bde|001\rangle)_{579}. \end{aligned} \quad (14)$$

According to measurement result of Alice, Bob operates a unitary transformation $(\sigma_x)_5 \otimes (\sigma_x)_7 \otimes I_9$, which will transform the state of particles (5, 7, 9) in (14) into $(-x^2y\alpha acf|000\rangle - xy^2\beta bde|111\rangle)_{579}$. To obtain the original unknown state, Bob introduces an auxiliary particle A with the initial state $|0\rangle_A$ and makes an unitary transformation U_1 under the basis $\{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}_{9A}$ [23]

$$U_1 = \begin{pmatrix} \frac{bdey}{acfx} & 0 & \sqrt{1-(\frac{bdey}{acfx})^2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \sqrt{1-(\frac{bdey}{acfx})^2} & 0 & -\frac{bdey}{acfx} & 0 \end{pmatrix}. \quad (15)$$

Then we get

$$\begin{aligned} &U_1(-x^2y\alpha acf|000\rangle - xy^2\beta bde|111\rangle)_{579} \otimes |0\rangle_A \\ &\quad = -xy^2bde(\alpha|000\rangle + \beta|111\rangle) \otimes |0\rangle_A \\ &\qquad - \sqrt{1-\left(\frac{bdey}{acfx}\right)^2}(x^2y\alpha acf)|001\rangle_{579} \otimes |1\rangle_A. \end{aligned} \quad (16)$$

Evidently, the maximal probability of successful teleportation is $|xy^2bde|^2$. After teleportation is finished, the next aim is to create the input state or its orthogonal state at Alice's place. Firstly, Alice sends particles 1, 2 and 3 to Victor. In the basis $\{|\varphi^i\rangle, |\varphi_\perp^i\rangle, (i = 0, 1, 2, 3)\}$ the entangled state $|\Phi_g^-\rangle_{14}|\Phi_g^+\rangle_{26}|\Psi_g^-\rangle_{38}$ can be rewritten as

$$\begin{aligned} &|\Phi_g^-\rangle_{14}|\Phi_g^+\rangle_{26}|\Psi_g^-\rangle_{38} \\ &\quad = |\varphi^0\rangle(-xy^2\beta^*|110\rangle - x^2y\alpha|001\rangle)_{468} + |\varphi_\perp^0\rangle(xy^2\alpha|110\rangle - x^2y\beta|001\rangle)_{468} \\ &\qquad + |\varphi^1\rangle(x^2y\beta^*|111\rangle + xy^2\alpha|000\rangle)_{468} + |\varphi_\perp^1\rangle(-x^2y\alpha|111\rangle + xy^2\beta|000\rangle)_{468} \\ &\qquad + |\varphi^2\rangle(-x^2y\beta^*|100\rangle - xy^2\alpha|011\rangle)_{468} + |\varphi_\perp^2\rangle(x^2y\alpha|100\rangle - xy^2\beta|011\rangle)_{468} \\ &\qquad + |\varphi^3\rangle(x^3\beta^*|101\rangle + y^3\alpha|010\rangle)_{468} + |\varphi_\perp^3\rangle(-x^3\alpha|101\rangle + y^3\beta|010\rangle)_{468}. \end{aligned} \quad (17)$$

Next, Victor performs a tripartite projective measurement on the particles 1. 2 and 3. If the result of Victor's measurement is $|\varphi_\perp^0\rangle_{123}$, the (14) can be written as

$$
\begin{aligned}
&|\varphi_\perp^0\rangle_{123}\langle\varphi_\perp^0|\Psi_g^-\rangle_{38}\langle\Psi_g^-|\Phi_g^+\rangle_{26}\langle\Phi_g^+|\Phi_g^-\rangle_{14}\langle\Phi_g^-|\Phi\rangle \\
&\quad= |\varphi_\perp^0\rangle_{123}(-x^2y^2)(y\alpha|110\rangle - x\beta|001\rangle)_{468}\otimes((\sigma_x)_5 \\
&\qquad\otimes(\sigma_x)_7\otimes I_9)(\alpha xacf|000\rangle+\beta ybde|111\rangle)_{579}. \qquad (18)
\end{aligned}
$$

Then, Victor sends the measurement outcome to Alice through a classical channel. According to the information of Victor, Alice knows that the state of her particles 4, 6 and 8 has been found in the state $(y\alpha|110\rangle - x\beta|001\rangle)_{468}$, Alice introduces an auxiliary particle B with the initial state $|0\rangle_B$ and makes an unitary transformation U_2 under the basis $\{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}_{8B}$ [23]

$$
U_2 = \begin{pmatrix} \frac{x}{y} & 0 & \sqrt{1-(\frac{x}{y})^2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \sqrt{1-(\frac{x}{y})^2} & 0 & -\frac{x}{y} & 0 \end{pmatrix}. \qquad (19)
$$

The unitary transformation U_2 will transform $(y\alpha|110\rangle - x\beta|001\rangle)_{468}\otimes|0\rangle_B$ into $x(\alpha|110\rangle - \beta|001\rangle)_{468}\otimes|0\rangle_B + \alpha y\sqrt{1-\frac{x^2}{y^2}}|111\rangle_{468}\otimes|1\rangle_B$. Alice can operate a unitary transformation $((\sigma_x)_4\otimes(i\sigma_y)_6\otimes I_8)$ to get a unknown state $(\alpha|000\rangle+\beta|111\rangle)_{468}$. Evidently, the maximal probability of successful teleportation is $|x^2y^2bde|^2$. As to other seven the measurement outcomes that Victor send to Alice through a classical channel, Alice can obtain a copy or a complement copy of the unknown state at her place (all are up to doing a rotation operation), respectively. In the process of teleportation, if the Alice's measurement results are other 63 entangled states, applying the similar analysis as above, Alice will obtain a copy or a complement copy of the unknown state at her place.

4 Conclusion

We have proposed a protocol which can produce perfect copies or orthogonal-complement copies of an unknown tripartite entangled state, via quantum and classical channel, with assistance of Victor (a state preparer). This protocol includes two stages. The first stage of the protocol requires usual teleportation. In the second stage, Victor will perform tripartite projective measurements on particles which from Alice. According to information from Victor, Alice can acquire either a perfect copy or an orthogonal-complement copy of unknown state. Furthermore, we have also considered that quantum channel is composed of three non-maximally entangled particle pairs. It is shown that for the non-maximally entangled quantum channel, Alice can produce the perfect copy or an orthogonal-complement copy of the input state with certain probability. As the last remark for our proposed protocol, the successful cloning is due to the intervention of the preparer of state. Incidentally, our present protocol of assisted cloning of an unknown three particle entangled state can be easily generalized to an unknown N-particle entangled state case.

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